**COSC 3304 – Algorithms Design and Analysis**

**Assignment 9**

**Due: 23:59:00pm, 04/04/2024**

1. The longest nondecreasing subsequence of a sequence is the longest subsequence in which all the elements are nondecreasing. This subsequence is not necessarily contiguous, or unique.

For example: ‘lamar’ has 3 longest nondecreasing subsequences, ‘lmr’, ‘amr’, and

‘aar’.

The longest common subsequence can be used to find the longest nondecreasing subsequence of a sequence.

* 1. The first sequence is the given original sequence,
  2. The second sequence is a sorted sequence by ordering all elements in the first sequence in nondecreasing/alphabetical order.
  3. Apply LCS to the first and second sequences to find the longest nondecreasing subsequence of the given original sequence (i.e., the first sequence).

Please find the longest nondecreasing subsequence of ‘algorithm’ using the steps above. (50 points)

1. Lets do the LCS using ALGORITHM and the letters put in alphabetical order:

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | A | L | G | O | R | I | T | H | M |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| A | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| G | 0 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| H | 0 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 3 | 3 |
| I | 0 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 3 | 3 |
| L | 0 | 1 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3 |
| M | 0 | 1 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 4 |
| O | 0 | 1 | 2 | 2 | 3 | 3 | 3 | 3 | 3 | 4 |
| R | 0 | 1 | 2 | 2 | 3 | 4 | 4 | 4 | 4 | 4 |
| T | 0 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 5 | 5 |

1. So we can see that the backward, the solution would be: “TROGA”
2. So the forward Longest Nondecreasing Subsequence would be “AGORT”

1. The change problem: The total amount is 28 and the denominations are 16, 13, and 1. How many bills will be used if applying a greedy algorithm? Is it the optimal solution? If not, what is the optimal solution? (20 points)
2. The greedy algorithm would take the most away first, so the order would be like:
   1. 28 – 16 = 12
   2. 12 – 1 – 1 – 1 – 1 (…) = 0
   3. That would be a total of 12 (1 dollar bills) + 1 (16 dollar bill) = 13 bills.
3. The dynamic method would be more like such:

A screenshot of a number

Description automatically generated

1. By this method, we can see that the optimal solution contains 4 bills: 2 13’s, and 2 1’s.

1. Please solve the fractional knapsack problem. What is the maximized benefit? (30 points)

4 items

Max weight is 8

Item 1: weight 5; benefit 7

Item 2: weight 2; benefit 4

Item 3: weight 3; benefit 5

Item 4: weight 4; benefit 6

* 1. Finding the benefit/weight ratio of each item:
     1. 1: 7/5 = 1.4
     2. 2: 4/2 = 2
     3. 3: 5/3 = 1.67
     4. 4: 6/4 = 1.5
  2. Taking the most valuable one would be Item 2 due to its ratio of 2. Adding all of Item 2, the bag will have a weight of 8 – 2 = 6.
  3. The second most valuable item would be Item 3 with a ratio of 1.67. Adding all of Item 3, the bag will have a weight of 6 – 3 = 3.
  4. The third most valuable item would be Item 4 with a ratio of 1.5. Adding Item 4 would not be valid as 3 – 4 = -1. Taking 3/4th of this item will be acceptable, as 0.75 \* 4 = 3, and using this weight, we would maximize the weight of the bag (3 – 3 = 0).
  5. In this scenario, Items 2, 3, and 3/4th of Item 4 makes a combined benefit of: 4 + 5 + (.75 \* 6) = 13.5